08

Year 12 Standard Mathematics

Module 08
Normal Distributions

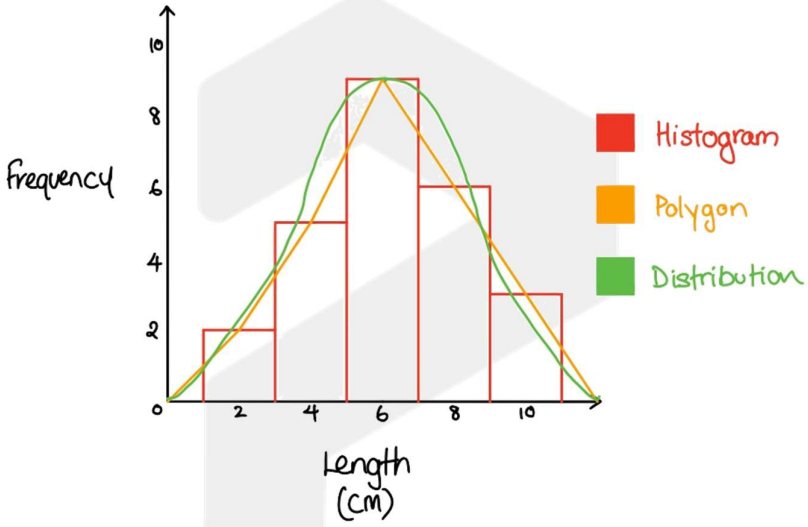
PROFECTUS

8.1 Frequency Distribution Graphs

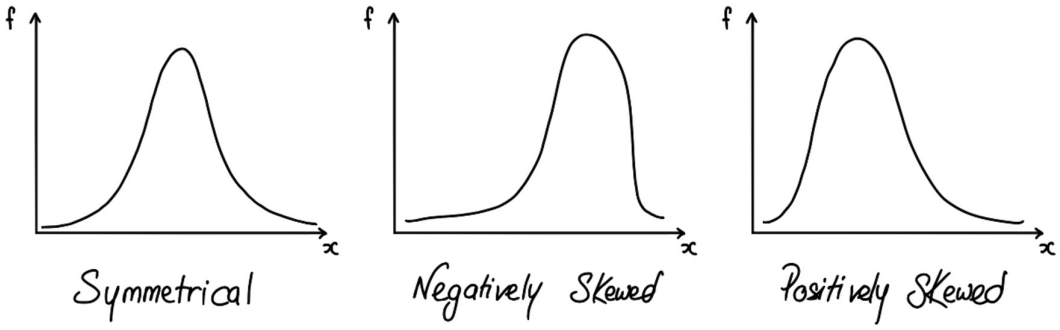
THEORY

- **What is a Frequency Distribution?** You may be familiar with a Frequency Histogram and Polygon. A Frequency Distribution is essentially when you:
    - Draw a smooth curved line over the histogram, OR
    - Smoothen the polygon into a curve

  See the diagram below.



- A Frequency Distribution can be:
    - **Symmetrical**: where the left half of the graph is identical to the right half of the graph.
    - **Skewed**: where the shape of the graph is not symmetrical.
        - **Positively Skewed**: where the graph thins out towards the right.
        - **Negatively Skewed**: where the graph thins out towards the left.



- **Recap of Mean, Median, Mode, Range**
  - **Mean**: The average i.e. $\frac{sum\ of\ all\ scores}{the\ number\ of\ scores}$
  - **Median**: The middle score
  - **Mode**: The most popular score
  - **Range**: The difference between the highest score and lowest score

WORKED EXAMPLES

1. On the same axes, draw a frequency histogram, a frequency polygon and a frequency distribution for the following data.

Mass (kg)	35	36	37	38	39	40
Frequency	7	10	16	11	8	4

   a. Comment on its skewness.
   b. Without calculating, comment on the mean and median. Which one do you expect to be higher?
   c. Find the mean, median and mode(s).

2. On the same axes, draw a frequency histogram, a frequency polygon and a frequency distribution for the following data.

Year	7	8	9	10	11	12
Frequency	70	80	140	160	170	90

   a. Comment on its skewness.
   b. Without calculating, comment on the mean and median. Which one do you expect to be higher?
   c. Find the mean, median and mode(s).

HOMEWORK PROBLEMS

Oxford Maths Standard 2, Exercise 9A, Page 381:

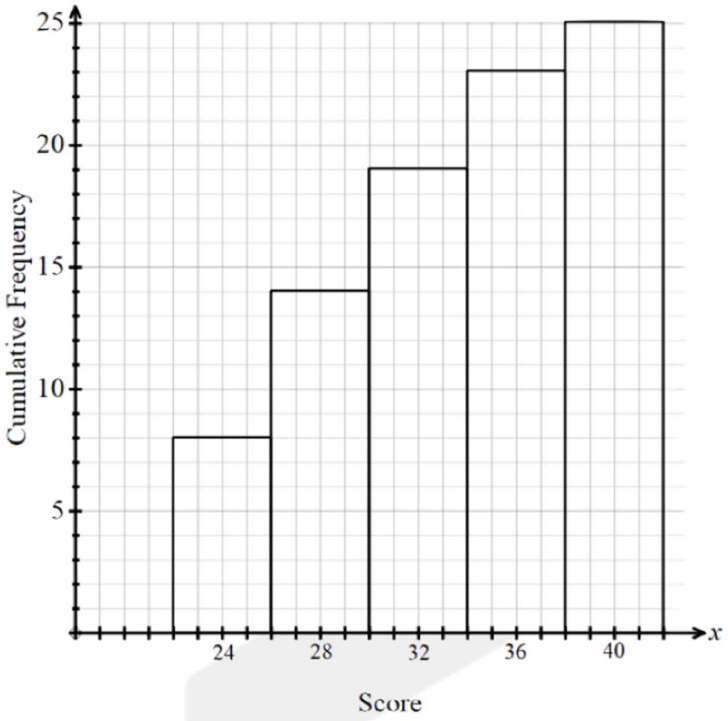
- Q1: e, f
- Q2: e, f
- Q3: e, f
- Q4: e, f
- Q5
- Q6

HARDER QUESTIONS

Question 1

Sam recorded the scores of 25 footballers who each took 50 shots at goal.

The grouped cumulative frequency graph displays the results with class centres of 24, 28, 32, 36 and 40.



(a) Use the graph to estimate the median number of goals scored.

(b) Calculate the mean number of goals scored. (Answer to 1 decimal place.)

(c) What percentage of players scored 37 goals or more?

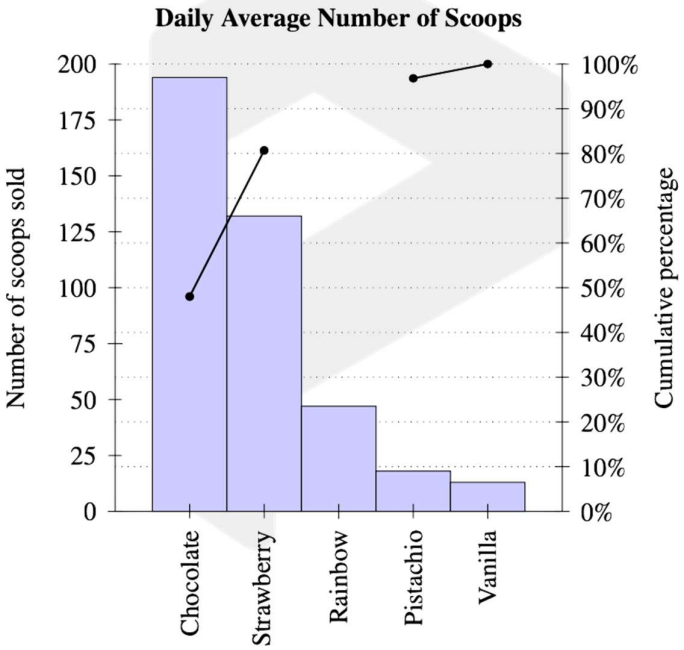
(d) Only players with scores in the top $76\,\%$ of all scores go through to the next round of shots. What score was needed to go through to the next round?

Question 2

Over the month of April, the local ice-cream shop recorded the average number of scoops sold per day for each of its flavours. The figures are shown in the table below.

Flavour	Cumulative Frequency
Chocolate	194
Strawberry	326
Rainbow	373
Pistachio	391
Vanilla	404

These results are also shown in the following Pareto chart.



(a) What was the average number of scoops of Strawberry ice-cream sold per day in April?

(b) Part of the line graph for the cumulative frequency is missing. Find the cumulative percentage for the Rainbow flavour. Give your answer correct to 1 decimal place.

(c) Using your answer to (b), complete the line graph above.

(d) As winter approaches and sales decline, the shop wants to reduce the amount of ice-cream in storage to prevent wastage. By applying the Pareto principle, which flavours would you advise the store to keep in stock. Justify your answer with appropriate reasoning.

Answers

Section 8.1: Frequency Distribution Graphs

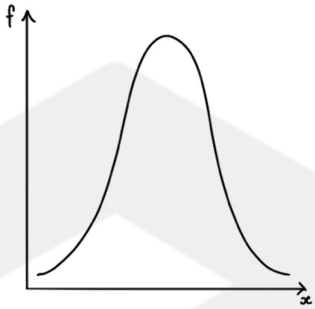
Q1:	(a) 29	(b) 29.8	(c) 16 %	(d) 25
Q2:	(a) 131	(b) 92.3 %	(c) Graph	(d) Explanation

8.2 The Normal Distribution

THEORY

- The **Normal Distribution** is a very, very special type of Frequency Distribution. It:
  - Is symmetrical at the mean
  - Has the same mean, mode and median
  - Has only one mode

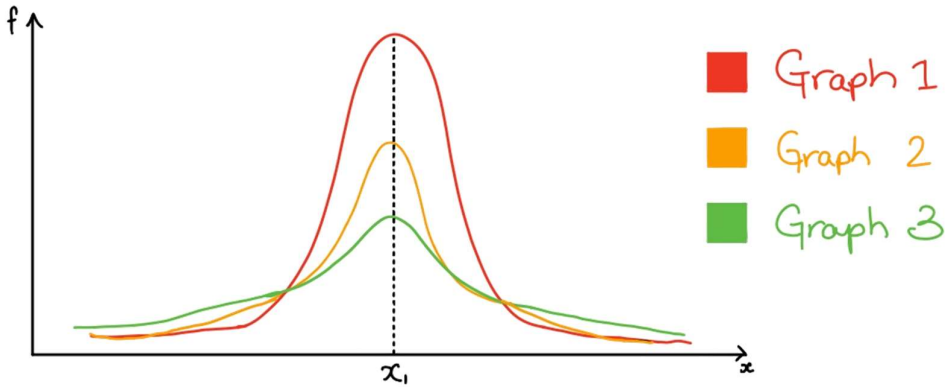
  It may also be called a 'Bell Curve' (because it's shaped like a bell) or a 'Gaussian Curve' (named after the famous mathematician who discovered it). See the graph below.



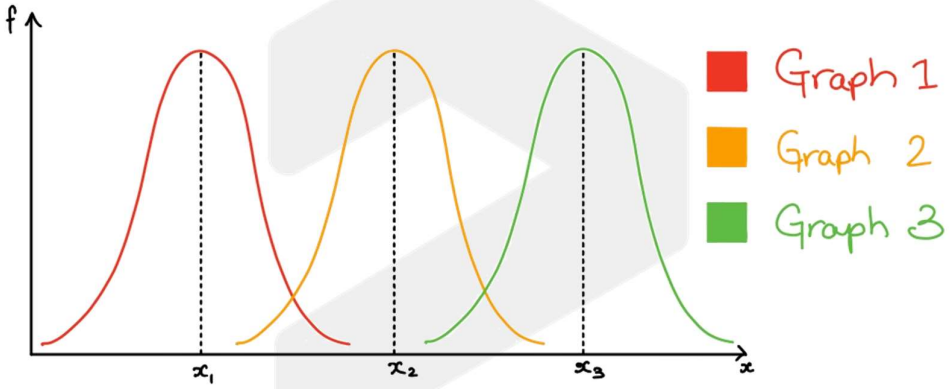
- It is one of the greatest discoveries in Math and certainly the **most powerful concept in statistics**. It is fascinating because most data sets collected from the natural world approach a Normal Distribution as you increase the sample size. This makes it an extremely powerful tool to create inferences (i.e. predictions from a data set). Here are some examples of data sets which would form a Normal Distribution:
  - Heights of people
  - Weights of people
  - Size of things produced by machines
  - Errors in measurements
  - Blood pressure
  - Marks on a test
  - Number of leaves on a tree
  - Numbers of hairs on a human head

    Etc.

- **Spread**. Normal Distributions can have varying levels of spread. The spread is measured by the standard deviation (more on this in the next chapter). See the figure below.
  - Graph 1 has the smallest standard deviation of the three graphs; the data set is very bunched together; it is not very spread out.
  - Graph 2 has a slightly bigger standard deviation; the data set is less bunch together; it is more spread out.
  - Graph 3 has the biggest standard deviation; the data set is least bunched together; it is more spread out.



- **Mean**. Normal Distributions can also have varying means. See the figure below.
    - Graph 1 has the smallest mean.
    - Graph 2 has a slightly bigger mean.
    - Graph 3 has the biggest mean.



- In this chapter, we only focus on identifying and reading Normal Distributions. In a future chapter we will look at the properties of a Normal Distribution and how to use these properties.

WORKED EXAMPLES

Determine whether the following data sets are Normal Distributions or not:

1.

Score	5	6	7	8	9	10	11
Frequency	1	7	14	18	13	5	2

2.

Score	5	6	7	8	9	10	11
Frequency	3	8	15	10	16	6	2

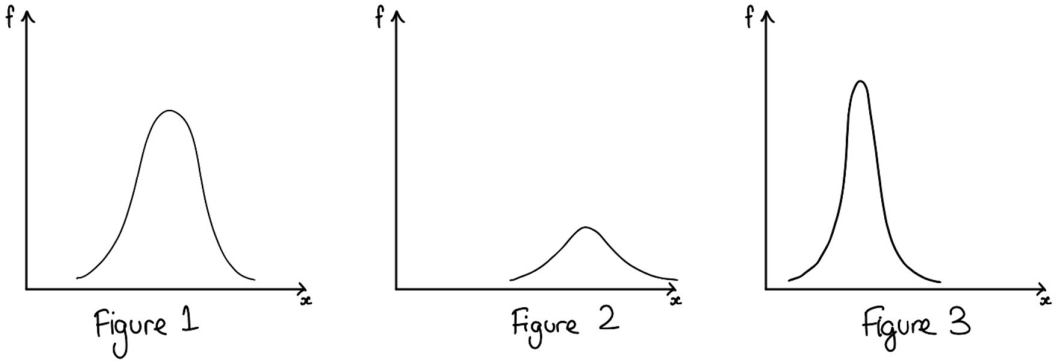
3.

Score	5	6	7	8	9	10	11

Frequency	3	12	18	31	19	13	1

WORKED EXAMPLES – ABSTRACT

The following Normal Distributions are drawn with the same x and f scale.



Which figure has the:

1. Largest standard deviation?
2. Smallest standard deviation?
3. Largest mean?
4. Smallest mean?

HOMEWORK PROBLEMS

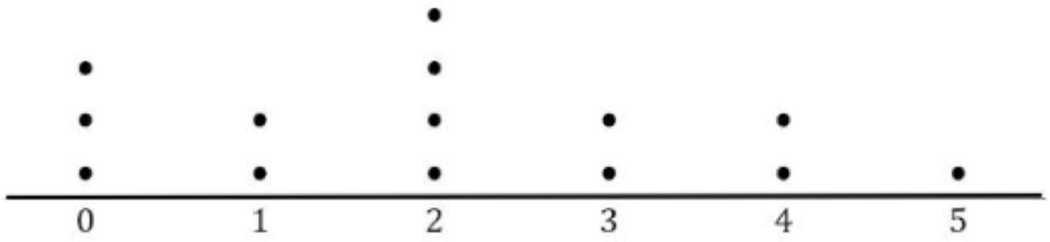
Oxford Maths Standard 2, Exercise 9B, Page 383 to 385:

- Q1
- Q2
- Q3: c, d
- Q4
- Q6
- Q7
- Q8

HARDER QUESTIONS

Question 1

14 people were asked to indicate the time (in hours) they had spent watching television on the previous night. The results are displayed in the dot plot below.



Find the mean and population standard deviation of these times. Correct your answers to one decimal place.

Question 2

Samuel scored $71\ \%$ in his first school assessment task for which the mean was $83\ \%$ and the standard deviation was 6. In his second assessment task, he scored $66\ \%$; for which the mean was $76\ \%$ and the standard deviation was 10.

Did Samuel's performance improve from his first assessment task to his second? Justify your answer with reference to Samuel's z-scores.

Answers

Section 8.2: The Normal Distribution

Q1: $\bar{x} = 2.1, \sigma = 1.5$

Q2: Explanation