04

Year 10 Mathematics

Linear Relationships and Number Plane

PROFECTUS

4.1 Revision:

Solving Linear Equations

THEORY

Rules:

- Whatever operation you do to one side, you MUST also do to the other side.
  - Note that we said "side", not "pronumeral" or "term". You either change the whole side or do nothing at all.
  - Operations can involve anything e.g. adding by a number, subtracting by a number, multiplying by a number, dividing by a number, square rooting, reciprocating, etc.

- Whatever pronumeral you are solving for (e.g. x, a, etc.), you must ensure that it only appears on one side and not both.

- Whatever pronumeral you are solving for (e.g. x, a, etc.), you must ensure that it is:
  - Positive (the positive sign will be invisible).
  - Singular (i.e. have a coefficient of 1, the 1 is invisible as well)
  - The ONLY thing in the entire side

- Whenever you see a fraction, get rid of it first. We can eliminate fractions by using the cross-multiply technique but note that this is not the only way. Also note that this particular rule may sometimes not be the fastest or most optimal way to arrive at the answer.

HARDER QUESTIONS

Question 1

Solve the following:

(a) $(x-3)(x+6)=(x-4)(x-5)$

(b) $(1+2x)(4+3x)=(2-x)(5-6x)$

(c) $(x+3)^2=(x-1)^2$

(d) $(2x-5)(2x+5)=(2x-3)^2$

Question 2

Solve the following equations involving fractions:

(a) $\dfrac{a+5}{2} - \dfrac{a-1}{3} = 1$

(b) $\dfrac{3}{4} - \dfrac{x+1}{12} = \dfrac{2}{3} - \dfrac{x-1}{6}$

(c) $\dfrac{3}{4}(x-1) - \dfrac{1}{2}(3x+2) = 0$

(d) $\dfrac{4x+1}{6} - \dfrac{2x-1}{15} = \dfrac{3x-5}{5} - \dfrac{6x+1}{10}$

Question 3

Solve for x:

(a) $\dfrac{x}{x-2} + \dfrac{3}{x-4} = 1$

(b) $\dfrac{3x-2}{2x-3} - \dfrac{x+17}{x+10} = \dfrac{1}{2}$

Question 4

(a) Show that $\dfrac{x-1}{x-3} = 1 + \dfrac{2}{x-3}$.

(b) Hence solve $\dfrac{x-1}{x-3} - \dfrac{x-3}{x-5} = \dfrac{x-5}{x-7} - \dfrac{x-7}{x-9}$.

Question 5

Two trains travel at speeds of 72 km/h and 48 km/h respectively. If they start at the same time and travel towards each other from two places 600 km apart, how long will it be before they meet? $\dfrac{x}{2} + \dfrac{y}{5} = 1$

Answers

Section 4.1: Solving Linear Equations

Q1: (a) $\frac{19}{6}$ (b) $\frac{3}{14}$ (c) -1 (d) $\frac{17}{6}$

Q2: (a) -11 (b) 2 (c) $-\frac{7}{3}$ (d) $-\frac{5}{2}$

Q3: (a) $\frac{14}{5}$ (b) 4

Q4: (a) Proof (b) 6

Q5: 5 hrs

4.2 Linear Inequalities

THEORY

The Inequality Signs

$>$ Greater than (open circle)

$\geq$ Greater than or equal to (closed circle)

$<$ Less than (open circle)

$\leq$ Less than or equal to (closed circle)

Rules and Notes

- The same rules apply as with normal equations. Pretend you are doing a normal equation with an equal sign, however;
- There are only 2 instances where you will need to flip the sign:
  - When you multiply or divide by a negative number
  - When you switch the sides
- At times you will be asked to draw the graph. See below.

Graphing an Inequality

- Once you have solved the inequation, draw a number line (just x-axis) and place your resulting number at the centre of this number line.
- Draw a small circle on your resulting number. If the inequality sign was $<$ or $>$ then leave your circle open; if the inequality sign was $\leq$ or $\geq$ then make your circle closed.
- From this circle, draw an arrow either going left or right as indicated by the inequality sign.

WORKED EXAMPLES

Solve the following inequalities and graph them on a number line:

1. $3x + 5 > 14$
2. $4 - \frac{x}{3} \leq 6$
3. $3x + 2 > 6x - 4$

HARDER QUESTIONS

Question 1

Solve the following inequalities and show the solution on a number line:

(a) $\frac{3x}{5} - \frac{2x}{3} > -2$

(b) $\frac{7x}{3} < 3 + \frac{4x}{3}$

(c) 19 $\frac{x-5}{2} > \frac{5x-3}{6}$

(d) 20 $\frac{5x-3}{2} < x+2$

(e) 21 $-5 < x+4 < 1$

(f) 22 $\leq 5x - 3 \leq 32$

(g) $-3 \leq \frac{2x-1}{3} < 3$

<u>Question 2</u>

If 5 is subtracted from a certain positive integer, the result is greater than 5 but less than 12. What values can this integer take? (Let the positive integer be x, so that $5 < x - 5 < 12$.)

<u>Question 3</u>

If a certain number is divided by 2, the result is greater than 4 but less than 8. What values can this number take?

<u>Question 4</u>

The sum of two consecutive positive integers is no more than 35. What are the possible values of these integers?

<u>Question 5</u>

A committee consists of 3 more women than men. The total number of committee members is at least 7 but not more than 15. How many women could be on the committee?

<u>Question 6</u>

When a certain number is added to 5 and the sum is divided by 5, the result is not more than if this same number is added to 13 and the sum is divided by 9. What is the largest value this number can take?

<u>Question 7</u>

The base length of an isosceles triangle is an integer (in cm) and is 4 cm less than the sum of the two equal sides. The perimeter is an integer (in cm) less than $80\,\mathrm{cm}$. What are the possible base lengths?

<u>Question 8</u>

The sum of three consecutive integers is greater than 7 and less than 25. Find all possible values of the smallest of these integers.

<u>Question 9</u>

The side lengths of a triangle are 8 cm, 10 cm and $x \text{ cm}$. What are the possible values of x?

Answers

Section 4.2: Linear Inequalities

Q1: (a) $x < 30$ (b) $x < 3$ (c) $x < -6$ (d) $x < \frac{7}{3}$
(e) $-9 < x < -3$ (f) $5 \leq x \leq 7$ (g) $-4 \leq x \leq 5$

Q2: $x = 11,12,13,14,15,16$

Q3: $8 < x < 16$

Q4: $x = 1,\ 2,\ \ldots,\ 17$

Q5: $5,\ 6,\ \ldots,\ 9$

Q6: $x = 5$

Q7: $0 < x < 38$

Q8: $x = 2,\ 3,\ \ldots,\ 7$

Q9: $2 < x < 18$

4.3 Graphing Lines

THEORY

- A linear function is essentially a **straight line**. Non-linear is a curved line (we'll look at this later!).

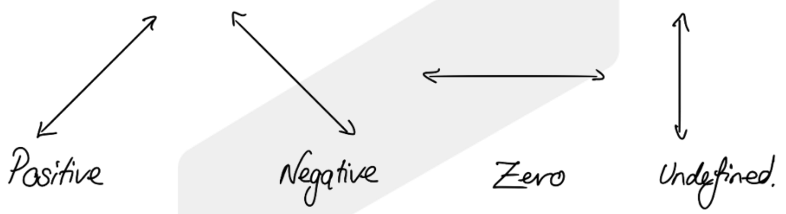
- Before learning how to graph linear functions, you must be familiar with **gradients** and **intercepts**.

- **What is a Gradient?**
    - It is a number that describes the slope of a line.
    - It is typically denoted by an 'm'.
    - Think of it as the change in y per 1 unit change in x.
    - Its formula is as follows:

$$gradient = \frac{rise}{run}$$

$$gradient = \frac{-}{\qquad}$$

    - Gradients can be:



    - The gradient can be calculated using the formula $m = \tan(\theta)$ where θ is the angle between the line and the positive x-axis.

- **What is an Intercept?**
    - There are 2 types of intercepts:
        1. y-intercept: Where the line touches the y-axis; occurs at $x = 0$
        2. x-intercept: Where the line touches the x-axis; occurs at $y = 0$

    - Important to note that an intercept is essentially a coordinate / point. This means it has an x-value and a y-value.

- To **graph** a linear function, we can use either the:
    1. Table of Values Method (aka the Dinosaur Method)
    2. Gradient-Intercept Method
    3. X and Y Intercept Method

- The **Gradient-Intercept Method** is used as follows:
    1. First ensure that the equation is in the form $y = mx + b$.
        - Note that y is the subject and has a coefficient of '$+1$'.
        - If it is not in this form, then convert it to this form.

2. b is your y-intercept. Plot this point on your graph.
3. m is your gradient. Continue the point you plotted in the previous step such that it reflects your gradient.
    - Keep in mind that your gradient will be either (1) positive or negative, and (2) steep or flat.
4. Keep the line going on both ends and insert arrows for continuity.

- The **X and Y Intercept Method** is used as follows:
    1. Find the x-intercept.
        - Let $y = 0$, then solve your equation for x.
    2. Find the y-intercept.
        - Let $x = 0$, then solve your equation for y.
    3. Plot both intercepts on your graph.
    4. Join the two dots with a straight line.
    5. Keep it going on both ends and insert arrows for continuity.

- The X and Y Intercept Method is generally quicker, but the Gradient-Intercept Method is more powerful.

WORKED EXAMPLES – SUBSTITUTION

Does the point $(-2, 7)$ lie on the following lines?

1. $y = -3x + 1$
2. $2x + 2y = 1$

WORKED EXAMPLES – GRAPHING LINES

Graph the following equations using the (a) gradient-intercept method and the (b) x and y intercept method:

3. $y = 2x + 3$
4. $y = 10x - 1$
5. $y = -x - 47$
6. $2y = -18x + 6$
7. $-4y = -16 + 64x$
8. $y = 3x$

WORKED EXAMPLES – UNDERSTANDING GRADIENT, X-INTERCEPT AND Y-INTERCEPT

For each graph drawn in the previous worked examples:

1. Indicate whether the slope is positive or negative.
2. Indicate whether the slope is steep or flat.
3. Circle the x-intercepts and the y-intercepts.
4. Find the y-coordinate when the x-coordinate is 3.
5. Find the x-coordinate when the y-coordinate is – 11.

WORKED EXAMPLES – UNUSUAL LINES

Graph the following equations and comment on their features:

1. $y = 5$
2. $y = -5$
3. $x = 5$
4. $x = -5$
5. $0 = x + 5$

HOMEWORK PROBLEMS

Cambridge Year 10 Math, Exercise 5E, Page 268 to 270:

- Q1
- Q2
- Q3
- Q4: d, e, f
- Q5
- Q6
- Q7
- Q8
- Q11
- Q13
- Q15
- Q16
- Q17: a, b

HARDER QUESTIONS

Question 1

(a) What is the natural domain of the relation $\dfrac{y}{x-1} = 3$?

(b) Graph this relation and indicate how it differs from the graph of the straight line $y = 3x - 3$.

Question 2

Find the point of intersection of $px + qy = 1$ and $qx + py = 1$, and explain why these lines intersect on the line $y = x$.

Question 3

Determine the equation of a line through $M(4, 3)$ if M is the midpoint of the intercepts on the x-axis and y-axis, and then sketch the line.

Question 4

$A(0, 0)$, $B(2, 1)$ and $C(1, 5)$ are the vertices of a triangle ABC. The triangle is rotated anticlockwise about the point A, through a right angle in the cartesian plane. Find the equation of the image lines of:

(a) the line AC

(b) the line AB

(c) the line BC.

Answers

Section 4.3: Graphing Lines

Q1: (a) $x \in \mathbb{R},\ x \neq 1$ (b) Graph

Q2: $\left(\dfrac{1}{p+q}, \dfrac{1}{p+q}\right)$, Explanation

Q3: $3x + 4y - 24 = 0$

Q4: (a) $x + 5y = 0$ (b) $2x + y = 0$ (c) $x - 4y + 9 = 0$